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Functional Polycentricity: A Formal Definition in Terms of Social Network Analysis

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Summary. Polycentricity is often used descriptively with regard to a regional system of settlements, usually referred to as a polycentric urban region (PUR). Although presented in much of the literature as in essence a morphological concept, polycentricity possesses a functional element that receives less attention. Polycentricity is also seen as a normative concept. However, it has not been rigorously defined using formal techniques. This paper argues that defining polycentricity in terms of both morphology and function is possible by drawing on techniques originating in social network analysis. The paper sets out a formal definition and derivation of functional polycentricity based on these techniques, which is then extended to a derivation of an index of regional functional polycentricity. The paper sets out worked examples to show how the techniques described might be utilised. The paper closes with a discussion of issues that may arise when putting these definitions into practice.

General Statement of the Problem

For all that polycentricity has become the focus of much spatial policy, it has about it a certain blurred, penumbral quality. Polycentricity, or polycentrism as it is sometimes called, can be found at the heart of planning policies at scales that go from the European (EC, 1999) to the regional (Government Office for the West Midlands, 2004). The European Spatial Development Perspective states with regard to the "concept of polycentric development" (itself not defined) that the

> creation of several dynamic zones of global economic integration, well distributed throughout the EU territory and comprising a network of internationally accessible metropolitan regions and their linked hinterland (towns, cities and rural areas of varying sizes), will play a key role in improving spatial balance in Europe (EC, 1999, p. 20).

The rise and rise of polycentricity reflects a growing consensus that spatial form is becoming increasingly dispersed; this consensus signals a more general shift in the way in which cities are conceptualised. Copus (2001) argues that this shift is from a Chicago school core–periphery model to more aspatial conceptualisations, while Batty has explored more general notions of the polynucleated urban landscape (Batty, 2001b) and small-world networks (Batty, 2001a) with regard to conceptualising urban form. But this general shift brings with it the problem,

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pointed out by Davoudi (2003), that although the general notion of polycentricity has been conceptualised as both normative and analytical, there has been little consistency in either how it has been treated, or how it has been defined. The primary focus of this article is therefore a formal method of defining polycentricity in terms of *functional* connections between settlements. This definition is then extended through the introduction of distance between settlements to provide the secondary focus: the formal definition of a polycentric urban region (PUR) in terms of both functional connections and distance between settlements. First of all, however, a brief history of polycentricity is in order. It comprises the origins of the concept, usage and definitions.

A Brief History of Polycentricity

Origins

In 2001, Michael Batty illustrated, using computer simulations, how a polynucleated spatial form could arise spontaneously, an emergent and path-dependent property of the ways in which people interact with one another (Batty, 2001b). Batty made another point too: the long historical provenance of networks of settlements, he argued, is something that has been largely overlooked in the current discourses on polycentricity (Batty, 2001b, p. 635). This, he maintained, is because the notion of a "deeper continuity" in the formation of settlements has simply "not been central to the theory of cities" (p. 635). The modern notion of polycentricity, like much else in planning history, can be traced back to the late 19th and early 20th centuries. In *Cities in Evolution*, Patrick Geddes used the phrase 'city-region' as a milestone *en route* to his coinage of 'conurbation' (Geddes, 1915/1968, p. 34). In so doing, Geddes also perceived the existence of what might now be called a polycentric urban region, even drawing the distinction, more recently made by Champion (2001), between a polynucleated urban region that has developed around a single metropolis and one that has come about through the functional coalescence of several equally sized settlements (Musterd and van Zelm, 2001). When discussing Greater London and south-east England, for example, Geddes set out a still-recognisable description, although the contemporary observer might add to the mix motorways and broadband Internet access

> Instead of the old lines of division we have new lines of union: the very word 'lines' nowadays most readily suggesting the railways, which are the throbbing arteries, the roaring pulses of the intensely living whole; or again, suggesting the telegraph wires beside them, so many nerves, each carrying impulses of idea and action either way (Geddes, 1915/1968, pp. 27–28).

While Geddes' description is basically an analytical one, we must turn to Lewis Mumford for the first use of the notion as a normative concept. In *The Culture of Cities* (1938), Mumford suggests that to "break up the functionless, overgrown urban masses of the past" a new type of city comprising a "cluster of communities, adequately spaced and bounded" is needed: this he calls the "poly-nucleated city" (Mumford, 1938, p. 489). As Hall (1984) notes, this is of course precisely what Ebenezer Howard proposed in the notion of the social city (Howard, 1898) and, indeed, Mumford notes that its principles are already well established in designs for Radburn, Köln and Hamburg (Mumford, 1938, p. 490). One could even make the admittedly invidious case that Mumford actually pre-empted Castells' theory of a network society (Castells, 2000) by 60 years, laying out a vision of a neotechnic society living in a

> Highwayless Town . . . in which the various functional parts of the structure are isolated topographically as urban zones (Mumford, 1938, p. 490)

although further discussion of that is inappropriate in the present paper. The point is that, even in terms of modern planning history, Batty (2001b) was right. Between the three of them, Howard, Geddes and

Mumford had, by 1938, laid out a strategy for analysis and policy, based around the general notion of polycentric development, that remains relevant to this day.

Use as a Normative and Analytical Concept

The relevance of polycentricity in the post-War context was highlighted by Hall in *The World Cities* (Hall, 1984), who suggested that the Randstadt, Holland, could be defined as a polycentric metropolis, by dint of the fact that it had no obvious primary settlement. Amsterdam, the Hague, Utrecht and Rotterdam together formed a single system with multiple foci which, it was hoped, would enable it to compete with cities such as London and New York (Lambregts and Zonneveld, 2004).

Since then, the general concept of polycentricity has been put to a wide variety of different uses. This may in part be attributed to the fact that, as we shall see in the next section, no precise, empirically testable definition of polycentricity has gained wide acceptance, even if there is a degree of consensus on what constitutes a polycentric urban region (Kloosterman and Musterd, 2001; Parr, 2004). Another clue lies in the frequent occurrence in the literature of the word 'polycentrism', which points to a normative interpretation rather than a more analytical one. It is indeed the case that, as presented in the ESDP (EC, 1999, p. 20), polycentricity is a stated objective. That it has become, for some, an 'ism' suggests that it has been accepted as a normative concept, even while the arguments as to its meaning continue. This has certainly been the case in the UK, where it has quickly become a central plank of regional and sub-regional spatial strategies (for example, DETR, 2002; Advantage West Midlands and East Midlands Development Agency, 2004; Government Office for the West Midlands, 2004; Northern Ireland Department for Regional Development, 2004; Scottish Executive, 2004). The problem, as Turok (2005) has observed, is that its usefulness in this context remains unproven, although evidence from across Europe suggests that it may have its advantages.

Meijers (2005), for example, explores the roles of two different types of network—'club networks', based around a common interest shared by the network's actors, and 'web networks', exemplified by a supply chain—in generating synergies in Randstad, Holland. Meijers finds that networks, and by extension polycentric systems, do indeed generate synergies, although those synergies based on co-operation between actors turn out to be easier to pinpoint than synergies based on complementarity of actors' functions (Meijers, 2005).

Governa and Salone (2005) explore the ways in which polycentrism has influenced approaches to spatial planning in Italy since the publication of the ESDP, noting that the development of networks between actors, as opposed to cities, has been instrumental in bolstering levels of social capital. What makes Italy's networks different from 'top–down' initiatives in France, they argue, is that the networks in Italy have developed as a consequence of historical processes, rather than through the deliberate pursuit of polycentrism. This echoes Batty's work on the spontaneous development of polycentricity referred to earlier (Batty, 2001b).

Bailey and Turok (2001), by contrast, sound a note of caution in a study of polycentricity in the context of Scotland's two largest cities, Edinburgh and Glasgow. The study asked two simple questions: do Edinburgh and Glasgow comprise a PUR at present; and should they? Traditionally competitive rather than co-operative, these two cities do indeed fulfil the most basic prerequisite for a polycentric urban region—spatial proximity—but do not yet have the necessary functional interconnectedness. With regard to the second question, Bailey and Turok are more equivocal, pointing out that mere size is not sufficient to compete on the global stage (Bailey and Turok, 2001, p. 699). The idea that a group of smaller cities can co-operate to compete with a single city of similar overall size, strongly promoted by the Dutch planning authorities in the 1950s and 1960s (Lambregts and Zonneveld, 2004), is simplistic in the view of Bailey and Turok (2001). However, as Meijers (2005) pointed out, synergies can still be had.

Champion (2001) offers a quite different perspective and makes the point that the role of changing demography in the formation of different types of urban and regional form has received insufficient attention (he describes his paper as a "marker challenging the research community" (p. 674)). We can see what Champion means if we look at Copus (2001). Copus argues that changes in information technology, transport and communication are likely to be the main drivers of a changing urban topography in the 21st century, noting only that European policy sees polycentric development as helping to avoid demographic concentration in core areas (Copus, 2001, p. 548). Champion's findings, although tentative, are interesting, because they go much farther than this: a PUR will tend to experience less of a retirement exodus than an otherwise similar monocentric urban region; households with children will prefer to live in a PUR; and childless working couples may have contributed to the growth of PURs (Champion, 2001). In other words, what we see here are the beginnings of a more general exploration of how an increasingly polycentric spatial form affects and is affected by changes in society.

The shift towards more polycentric patterns of spatial development is not necessarily beneficial, though. In a study of commuting behaviour in the three largest French metropolitan areas, Aguilera (2005) found that polycentric systems of sub-centres tended to increase average commuting distances, as increasing numbers of people commute between sub-centres, rather than commuting within their own sub-centre. At a time when climate change is a pressing issue for anyone involved in spatial planning, we might observe that Aguilera's findings echo those of Breheny who, in analyses of live-work patterns in the UK, found that edge-to-edge commuting was increasing and was increasingly reliant on the use of the motor car (Breheny, 1997; Breheny and Hall, 1999).

Business relations have been explored through the lens of polycentricity, and in some detail, within the EU-funded POLYNET project (Hall and Pain, 2006a). Hall and colleagues analysed eight European 'polycentric mega-city regions' (South East England; Paris-Île de France; northern Switzerland; Randstad, Holland; the Dublin region, Ireland; Rhein–Main and Rhine–Rühr, Germany; and the Brussels region, Belgium) with a view to gaining a clearer understanding of how these immensely complex spatial systems function in terms of finance and business services. Here, polycentricity was used as an analytical concept rather than a normative one and it was in the course of this project that the notion of functional polycentricity was developed and tested empirically (Green, 2004; Hall *et al.*, 2006a). A more developed version of this is presented in the current paper.

Drawing on the findings of the POLYNET project, Pain (2006) argued that attempts to develop policies around polycentric development would create tensions between the need for balanced spatial development and the need for environmentally sustainable development, while in closing *The Polycentric Metropolis*, Hall and Pain (2006b) concluded, amongst other things, that there is still no clear understanding of whether or not polycentricity is either sustainable or economically competitive. They also concluded (of relevance to the current article) that the notion of functional polycentricity is less well understood than that of morphological polycentricity.

Moving still farther away from the notion of polycentricity as a normative concept, to its use as purely analytical concept, we find authors such as Anas and Kim (1996) who argue that traditional economic models of growth based on a monocentric model had been pursued for their simplicity rather than their accuracy. Anas and Kim proposed a general equilibrium model based instead on a polycentric topography that would, when compared with traditional monocentric models, better reflect the realities of how people use different employment centres (Anas and Kim, 1996).

The present article does not intend to comment on the merits or otherwise of 'polycentrism' as a general concept, but it seems clear from the literature that polycentricity is happening, sometimes as a consequence of policy, sometimes as a consequence of social change, and that it has both positive and negative aspects and that there is no real consensus over what it actually means. The problem, as we shall see in the next section, is this: While narrative definitions can be found easily enough and tend to be reasonably consistent with one another, the formal definition of both polycentricity and the polycentric urban region is a task that remains to be done.

Definitions: Polycentricity and Polycentric Urban Regions

When the European Spatial Development Strategy of 1999 (EC, 1999) placed polycentricity firmly at the heart of current spatial planning policies across Europe, primarily as a normative concept, a crucial question was raised: if polycentricity is an intended goal, how are we to recognise it?

One way of dealing with this question is to explore polycentricity through the systems it is intended to influence—that is, through the notion of the polycentric urban region (PUR). In effect, this means defining polycentricity itself primarily in terms of spatial organisation, but with some reference to functional interconnectedness (Ipenburg and Lambregts, 2001; Kloosterman and Musterd, 2001; ESPON, 2003). Kloosterman and Musterd, noting that "polycentricity can, in principle, refer to any clustering of human activity" (Kloosterman and Musterd, 2001, p. 623) offer a list of characteristics, summarised here, that a polycentric urban configuration can be assumed to possess (Kloosterman and Musterd, 2001, p. 628)

—a number of historically distinct cities;
—no obvious leading city;
—a small number of larger cities of similar size to one another, together with a greater number of smaller cities;
—constituent cities located relatively close to one another (within maximum commuting distance is suggested);
—constituent cities that are spatially and politically distinct from one another.

Parr (2004) suggests a framework for defining a polycentric urban region which is very similar to that proposed by Kloosterman and Musterd (2001). However, Parr does not define polycentricity itself either, except to note that "clearly, 'polycentric' refers to the plurality of centres" (Parr, 2004, p. 232). Parr defines the polycentric urban region as a cluster of discrete similarly sized settlements, separated by open tracts of land, with above-average (relative to a baseline region) interaction between them and each having a specialised economic structure (Parr, 2004).

Champion (2001, p. 664) takes a slightly different tack, suggesting that there are three basic definitions of a polycentric urban region, varying in how restrictive they are

—a collection of settlements in a region (least restrictive);
—as above, but with some interaction between settlements;
—as above, but each centre has a specialist function within the region (most restrictive).

This 'banding' complements the prerequisites laid down by Kloosterman and Musterd (2001) and Parr (2004) remarkably well: taken together, Kloosterman and Musterd's and Parr's prerequisites combine with Champion's 'bands' to produce quite a sophisticated definition of the polycentric urban region.

The most precise definition to date of polycentricity within a polycentric urban system is that offered by Spiekermann and Wegener (2004), who have developed a formal definition of polycentricity based on the rank–size distribution of settlements in an urban system. They suggest that polycentricity can be measured so that four basic requirements are met

—In a polycentric urban system, there is a distribution of large and small cities.

—In a polycentric urban system, the rank–size distribution is log-linear.
—A flat rank–size distribution is more polycentric than a steep one.
—A polycentric urban system is not dominated by one large city.

The rank–size distribution is based on the rank–size rule, which states that if settlements in a country are ranked in order of population size, then the population of a settlement ranked n will be $1/n$ th the size of the largest settlement (Mayhew, 1997). One issue that arises when defining polycentricity using the rank–size distribution is that in a system of settlements of identical size, there is no way of ranking them, except by assigning an arbitrary order to the settlements to generate a straight horizontal line. Admittedly, the Spiekermann–Wegener definition would not be satisfied in terms of its own Rule 1 if this were the case. However, it can be argued that, if we have to resort to assigning an arbitrary order in a particular theoretical case, then we have in effect pushed this particular approach to defining polycentricity to breaking-point, albeit under extreme theoretical conditions. Ideally, we want a theory that can remain intact when pushed to these extremes. It should also be noted that this approach takes no account of functional linkages between settlements. The Spiekermann–Wegener approach might therefore best be thought of as a means of gauging the relative extents of morphological polycentricity in a number of different urban systems when polycentricity is considered primarily in terms of settlement size. Indeed, Spiekermann and Wegener have used their approach to compare polycentricity in a number of different countries (Spiekermann and Wegener, 2004).

We can see that polycentricity has typically been situated within a regional context and has been conceptualised in largely morphological terms, although functional connections tend to be assumed. We have also seen that common themes emerge with regard to the attributes that polycentricity might be expected to possess and that they provide a good basis on which to develop a more formal definition of the kind set out in this article. In essence, polycentricity as it is envisaged in the ESDP and elsewhere can be seen as a combination of spatial topography and interconnectedness: a polycentric region might be conceived of as a network of settlements, although it is important to note that polycentricity makes no specific reference to scale, even if many commentators do (the scale being the region). However, the notion that polycentricity is, or at least should be, a scalable concept is implicit in the ESDP's statement and elsewhere is made explicit (see for example, Nadin and Dühr, 2005). This suggests that any formal definition of polycentricity should likewise be scalable. Lastly, in view of the 'fuzziness' that the concept of polycentricity appears to have, one can make the case that any attempt to quantify polycentricity should present it as a property that exists, as it were, on a sliding scale, rather than being a property that an urban system either does or does not possess. To put it very crudely, any group of reasonably closely spaced settlements is likely to be polycentric to some extent and any formal definition of polycentricity should reflect that.

Functional Polycentricity Defined

The approach presented in this paper is based on formal network analysis techniques (see Wasserman and Faust, 1997, for a thorough treatment) and refined from earlier versions which have been tested 'in the field' (Green, 2004, 2005; Hall *et al.*, 2006a). Briefly, networks comprise actors and linkages between them (these are referred to in graph theory as nodes and edges respectively). Actors may be cities, people, businesses, charities, telephones, computers: in short, anything capable of being connected to something else. Relations take the form of linkages: roads, friendships, telephone lines and business partnerships are all examples of linkages.

The relations between actors are thus functional in nature and, if we return to the notion set out above—that polycentricity as it is envisaged in the ESDP is a combination of spatial topography and interconnectedness—we can argue that, while the word 'polycentricity' addresses spatial topography, it does not address function. A qualifying term, 'functional' is therefore added to create the more specific term, 'functional polycentricity'. The approach set out here is therefore one that enables consistent analysis of polycentricity at any scale and for which spatial proximity is not a necessary condition. Note that this approach to defining polycentricity does not contradict the approaches described above. It is an alternative means of defining polycentricity to the Spiekermann–Wegener approach (Spiekermann and Wegener, 2004), while Champion (2001), Kloosterman and Musterd (2001) and Parr (2004), it will be recalled, proposed definitions for a polycentric urban region, rather than for polycentricity itself. Lastly, and importantly, this definition of polycentricity is qualified by the prefix 'functional'.

As stated earlier, a system of settlements can be conceived of as a network. At a general level, networks can be more or less dense and, of course, physical spaces vary both in size and in the number of settlements they contain. More specifically, there is not just one kind of network. Networks comprise actors and relations between those actors and may in consequence be 'trans-spatial': that is, the locations across space of those actors can change without altering the topology of the social network. For example, a group of friends remains a group of friends, no matter where each of those friends is physically located. Thus the physical shape of the network of three close friends—its topography—may change. For example, if they meet in a restaurant it will become smaller and then it will 'stretch out' again when they go to their different homes; however, its mathematical shape—its topology—remains constant (an equilateral triangle, in this case). In other words, topography may change, even as topology remains constant (Figure 1; also see Battersby, 2006, for a lay-person's introduction to multidimensional topology).

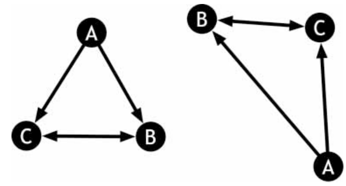

Figure 1. Topology and topography: each of the diagrams shows the same network of financial relations between three people, A, B and C. The arrows denote 'owes money to' (thus A owes money to both B and C). The thickness of the arrow reflects the amount of money owed (amounts owed are all the same in this instance). In the diagram on the left, the nodes are set out in an equilateral triangle and have been laid out so that nodes having the same line thickness are the same distance apart. In other words, they have been laid out topologically (in accordance with their mathematical relationships with one another). In the diagram on the right, the nodes have been laid out to reflect (say) where each of A, B and C lives. In other words, they have been laid out topographically. Note that the network's topology is unchanged in the diagram on the right.

Equally, when looking at functional relationships between cities, the networks clearly have a fixed physical shape, which is dependent upon the locations of those cities. However, the mathematical shape of a network is *not* intrinsically dependent upon its physical shape. Thus, for a particular function such as financial transactions, a group of cities can be close in network terms, while being physically distant, or they could be close in physical terms, while being distant in network terms. In terms of certain other functions, such as commuting, we might expect a correlation between physical and network proximity, but this correlation does not necessarily imply dependence.

To give a brief example of how the notion of functional connectivity can be brought to bear on the concept of polycentricity, we might consider the example of Tokyo, Hong Kong, Frankfurt, London and New York. These cities are highly connected in terms of financial transactions and can be said to form a network, although they are not spatially proximate. Using the techniques set out in this paper, it would be possible to compare the functional polycentricity of this system of five cities in terms not only of financial transactions, but also business trips and, say, friendships.

In the next section, then, we begin by looking at what basic ground rules are required, before moving on to derive a formal definition of polycentricity using techniques from social network analysis.

Basic Rules for Functional Polycentricity

From the discussion thus far, it follows that, for functional polycentricity to exist in any physical space, be it local, regional, national or global

—The space in question must contain more than one node (Rule 1)
—Those nodes must be functionally linked to one another, so that if no functional connections exist between nodes, then functional polycentricity cannot be said to exist (Rule 2).

Without both of these requirements being fulfilled, a group of nodes cannot be said to have functional polycentricity. We can thus assign a range of possible values to polycentricity. Formally, let P_F be polycentricity for function F, and assign to polycentricity a minimum value 0 (no functional polycentricity) and maximum value 1 (perfect functional polycentricity). Thus

$$0 \leq P_F \leq 1 \tag{1}$$

The two rules give us parameters within which to develop the definition. First, we need the basic formal definitions for a network comprising nodes connected by functional linkages. Let $N_F = \{n_1, \quad n_2, \ldots n_g\}$ be a functional network of g nodes and let $L = \{l_1, l_2, \ldots l_g\}$ be the set of lines between them denoting linkages for function F.

In a network, the more evenly distributed the functional linkages, the less likely it is that any single node will stand out from the others. Thus in a network that has a perfectly even distribution of linkages, there will be no node that is better connected than any of the others.

It is possible to measure the distribution of linkages by analysing nodal degree within the network and generating a ratio of actual variance of nodal degree within the network to a notional maximum variance, as suggested by Snijders (Wasserman and Faust, 1997, p. 181). This gives a value of 0 for a completely regular graph. However, with very large values of nodal degree, using the variance becomes problematic, since variance scales exponentially. An alternative is to use the standard deviation, which scales linearly and thus avoids this problem. Formally

$$P(F, N) = 1 - \sigma_F / \sigma_{F\max} \tag{2}$$

where, $P(F, N)$ is Ordinary Polycentricity with regard to a particular network relation F in a network N, such that 1 indicates complete monocentricity and 0 indicates complete Ordinary Polycentricity; σ_F is the standard deviation of nodal degree being measured; and $\sigma_{F\max}$ is the standard deviation of a 2-node network N_z where degree $n_1 = 0$ and degree $n_2 =$ degree of the node with the highest degree value in network N.

Since a standard deviation ratio of 0 implies complete Ordinary Polycentricity, we subtract from 1 so that complete polycentricity is assigned a value of 1 (satisfying equation (1)). Dividing standard deviation by a notional maximum possible standard deviation for the network is a means of normalising the index of standard deviation.

This topological definition satisfies only Rule 1: a maximally connected network and an unconnected network would both be completely polycentric since, mathematically speaking, each of the nodes is equally connected to each of the other nodes. However, three unconnected nodes do not comprise a single system and since we are concerned here with functional polycentricity we need to introduce the notion of network density—the ratio of actual connections to total potential connections—so that unconnected collections of nodes are not defined as a functionally polycentric system.

Defining Network Density

To meet the requirement of Rule 2—that functional polycentricity fall to zero where the number of linkages is zero—we must also measure the density of the graph Δ such that

$$0 \leq \Delta \leq 1 \tag{3}$$

There are four different ways of deriving a value for Δ depending on the type of relation (Wasserman and Faust, 1997, pp. 101–103, 129).

For a directed graph having dichotomous relations

$$\Delta = \frac{L}{g(g-1)} \tag{4}$$

For an undirected graph having dichotomous relations

$$\Delta = \frac{2L}{g(g-1)} \tag{5}$$

For a directed graph having valued relations (this is the most likely value in the context of this article)

$$\Delta = \frac{L}{L_{\max}} \tag{6}$$

For an undirected graph having valued relations

$$\Delta = \frac{2L}{L_{\max}} \tag{7}$$

where, Δ is density; g is the number of nodes in the graph; L is the number of edges in the graph; and $L_{\max}$ is the maximum number of edges in the graph that is theoretically possible.

$L_{\max}$ is not always easy to derive. For example, in a commuting network, there is a finite number of people who can commute, but the limits on numbers of e-mails, telephone calls or business meetings are more difficult to determine. In such cases, it is necessary to develop an empirically based theoretical maximum so that the requirement that $0 \leq \Delta \leq 1$ is met for equations (6) and (7). Clearly, this is a matter of judgement, but in many cases there will be a practical limit that is plausible. In the case of e-mail, for example, the upper limit of how many e-mails someone can send in a day (assuming they are not bulk e-mails) is bounded by the length and size of those e-mails and the speed of the computer and Internet connection. Nonetheless, even a careful calculation based on such parameters would not address issues such as the quality of communication embodied in each message. (Is an exchange of several e-mails to arrange a meeting equivalent to one telephone call that serves the same purpose?)

Of paramount importance, then, is that the basis for the final value for $L_{\max}$ should be set out clearly and that in comparisons of the same function in different regions, the same basis is used throughout for each function. In this way, the basis for the final value can be argued from a position of transparency and like can be compared with like. In other words, although the basis on which $L_{\max}$ is calculated can be criticised, the method remains internally consistent.

Special Functional Polycentricity

We are now in a position where, by multiplying equation (2) by density, we can derive a value for functional polycentricity for a single function. This we call Special Functional Polycentricity. Formally, let

$$P_{SF}(N) = \left(\frac{1-\sigma_{\partial}}{\sigma_{\partial\max}}\right) \cdot \Delta \tag{8}$$

where, P_{SF} is Special Functional Polycentricity for a function F within network N; σ_{∂} is the standard deviation of nodal degree; $\sigma_{\partial\max}$ is the standard deviation of the nodal degree of a 2-node network (n_1, n_2) derived from N where $d_{n1} = 0$ and d_{n2} = value of the node with highest value in N; and Δ is the density of the network.

The definition of Special Functional Polycentricity has three main facets. First, polycentricity is defined as a network theoretic function and so does not consider physical distances between nodes. This makes it scalable. Secondly, the definition includes density of the network which means that the level of interaction between places in terms of a particular function is taken into consideration and thus has a bearing on the extent to which a network of places may be considered to be a single system. Lastly, it defines polycentricity in functional terms and can thus be used to describe polycentricity across a variety of functions within the same geographical area. That is to say, for a given set of nodes, a variety of different functional relations can be measured and compared. For example, in-commuting, out-commuting, e-mail traffic, telephone conversations, leisure-related travel or the movement of money (to name just a few) can be measured and compared using the same network analytical techniques. In this way, the many different levels and types of polycentricity within a region can be analysed and compared, and a more layered and sophisticated analytical model of polycentricity developed.

General Functional Polycentricity

This section sets out a simple technique for combining several values of Special Functional Polycentricity into a single figure, which we call General Functional Polycentricity. For example, we may have values of Special Functional Polycentricity for financial transactions, commuting and leisure trips among several similarly sized towns in a region and wish to generate a more general value for functional polycentricity in terms of these three functions, so that we can compare the overall picture with that of three similar towns in a different region. This figure can be called General Functional Polycentricity, since it is not specific to a function. One way of doing this is simply to calculate the mean polycentricity for the different functions specified, which would give the equation

$$P_{GF}(N_1, N_2, \ldots N_n) = \frac{\sum_{n=1}^{n} P_{SF}(N_1, N_2, \ldots N_n)}{n} \tag{9}$$

where, P_{GF} $(N_1, N_2, \ldots N_n)$ is general functional polycentricity for functional networks $N_1, N_2, \ldots N_n$; P_{SF} $(N_1, N_2 \ldots N_n)$ are values of Special Functional Polycentricity for networks $N_1, N_2, \ldots N_n$; and n is the number of networks.

However, this equation is not capable of dealing with the special case that arises when we are dealing with a system that actually comprises multiple monocentric networks. In a three-node network, for example, each centre may be dominant for a particular function—say work, leisure and residential—and hence each functional network is more monocentric than polycentric. However, and as explored by Meijers (2005) with regard to the Randstad in the Netherlands, the functionally monocentric networks may complement one another, thus creating a functionally polycentric system when taken together.

To deal with this situation, it is necessary to introduce what may be called a complementarity modifier. This has the effect of weighting the index for General Functional Polycentricity so that the balance of each functional network relative to the others is taken into consideration. Thus networks having a similar balance will modify the index for General Functional Polycentricity less than networks that are not balanced relative to one another. The assumption here is that if the individual functional networks are about equally monocentric, then the system as a whole will be relatively balanced and therefore relatively polycentric. The formal derivation of this modifier is a two-step process

Step 1 calculate how balanced are the individual monocentric networks.

Step 2 normalise this figure so that, when multiplied by the modifier, a perfectly balanced system of monocentric networks does not change the value for P_{GF} but, as imbalance increases, P_{GF} is systematically reduced.

Using this approach, a well-balanced system of multiple monocentric networks will have a relatively high index of General Functional Polycentricity, reflecting the (mathematically) balanced nature of the system as a whole.

This can be formalised by calculating the standard deviation of the values for Ordinary Polycentricity (equation (2)) for each of the constituent networks (to continue using the example given earlier, these would be financial transactions, commuting and leisure trips) and using this as a multiplier. Formally, let

$$\Phi = 1 - \sigma_{P(F, N_1, \dots N_n)} \qquad (10)$$

where, Φ is a complementarity modifier and $0 \leq \Phi \leq 1$; $\sigma_{P(F, N_1, \dots N_n)}$ is the standard deviation of values for Ordinary Polycentricity, $P(F, N_1 \dots N_n)$ for functional networks $(N_1 \dots N_n)$ (defined in equation (2)).

We subtract from 1 so that the value for Φ is one when the networks are perfectly balanced and reduces as the networks become less balanced. Note that the minimum value is in fact one minus the standard deviation of one and zero ($1 - 0.71 = 0.29$).

This gives us a definition of General Functional Polycentricity that can handle systems comprising multiple monocentric networks. Formally

$$P_{GF}(N_1, N_2, \dots N_n) = \frac{\sum_{n=1}^{n}[P_{SF}(N_1, N_2, \dots N_n)]}{n} \cdot \Phi \qquad (11)$$

where, $\sum_{n=1}^{n}[P_{SF}(N_1, N_2, \dots N_n)]/n$ is General Functional Polycentricity for functional networks $N_1, N_2 \dots N_n$ as defined in equation (9); and Φ is a complementarity modifier as defined in equation (10).

Functionally Polycentric Urban Regions

We can use a similar approach to define tentatively an index of Regional Functional Polycentricity. In principle, this involves taking the ground rules laid out in particular by Kloosterman and Musterd (2001) and Parr (2004) and placing them within a formal framework.

The basic assumptions are that

—Settlements deemed to be within a polycentric urban region are within a defined range of distances from one another (following Kloosterman and Musterd, 2001; Parr, 2004) (Rule PUR 1).

—The settlements within a polycentric urban region are functionally interconnected (following rule 2 above) (Rule PUR 2).

—The settlements within a polycentric urban region will be of approximately similar size (again following Kloosterman and Musterd, 2001; Parr, 2004) (Rule PUR 3).

What we are exploring here is topographical balance within the network and we can therefore use the equation of Ordinary Polycentricity (equation (2)) as the basis for a formal definition of an index of Regional Functional Polycentricity.

First, we must develop a formal means of specifying the range of distances between centres. Thus, let d_{ij} be the distance between two settlements of similar size, i and j. For a polycentric urban region (following Rule PUR 1) let

$$0 \leq d_{ij} \leq d_{ij(\max)} \qquad (12)$$

where, $d_{ij(\max)}$ is determined according to the mean distance between settlements $\overline{D}$ such that

$$d_{ij(\max)} = \overline{D} + \sigma_T \qquad (13)$$

where, $\overline{D}$ is the mean distance between settlements; and σ_T is the standard deviation of all d_{ij} for the system of settlements.

Note that $\overline{D}$ need not be measured in terms of physical distance. Time (travel-to-work time, for example) may be an equally valid measurement of distance. Note also that the specification $d_{ij(\max)}$ as being one standard deviation greater than the mean is suggested as a starting-point that may be varied according to context. An alternative approach, for example, could be to follow Geddes' observation that one hour is about the maximum tolerable commute time (Geddes, 1915/1968, p. 41) and to set the maximum distance $d_{ij(\max)}$ as one hour in a particular mode of transport. It may also be the case that we are trying to calculate the index of Regional Functional Polycentricity for a predetermined collection of settlements, in which case $d_{ij(\max)}$ is already fixed.

Equation (13), then, offers a means of calculating $d_{ij(\max)}$ in the absence of any other information. Let the index of Regional Functional Polycentricity be defined either in terms of Special Functional Polycentricity or General Functional Polycentricity (following Rule PUR 2). The approach here is to argue that a PUR comprising settlements that are all equidistant from one another (note that only the points of an equilateral triangle, or any two points, fit this requirement) would not have its value for functional polycentricity altered. However, as the settlements become less evenly distributed in terms of distance from one another, so the PUR becomes progressively less topographically polycentric. This echoes the notion of a 'Connnectivity Index' derived in the ESPON project, which is a measure of potential accessibility of a node (Nordregio, 2005). To formalise this, we can modify the equation for Ordinary Polycentricity (equation (2)) and use it to measure the Ordinary Polycentricity of a PUR in terms of distance between settlements.

Thus let

$$P_T = 1 - \sigma_T/\sigma_{T\max} \quad (14)$$

where, P_T is Topographical Polycentricity; σ_T is the standard deviation of all d_{ij} for a system of settlements all of which are within the range $0 \leq d_{ij} \leq d_{ij(\max)}$, where $d_{ij(\max)}$ is calculated either following equation (13) or in an alternative manner as set out in the discussion immediately following the derivation of equation (13); and $\sigma_{T\max}$ is the standard deviation of $\{d_{ij} \ldots d_{nm}\}$ where $d_{ij} = 0$ and d_{nm} is the distance between the two most widely spaced settlements in the system.

We can use P_T as multiplier and thus we can say that

$$R_{SF} = P_T \cdot P_{SF} \quad (15)$$

and

$$R_{GF} = P_T \cdot P_{GF} \quad (16)$$

where, R_{SF} is the index of Regional Functional Polycentricity for a single function F; R_{GF} is the index of Regional Functional Polycentricity for more than one function $F_1 \ldots F_N$; P_T is Topographical Polycentricity; P_{SF} is Special Functional Polycentricity; and P_{GF} is General Functional Polycentricity.

Application in Practice

Choice of Scale and Settlements

The definitions set out thus far enable the definition of the polycentricity of a network of settlements in terms of functional connections between them, distance between them (measured in either time or distance), or some combination of these. The definitions of functional polycentricity can be applied to settlements having any spatial distribution: towards one extreme might be rooms in an office building, towards another extreme might be capital cities across the globe. This is important because a given space may contain both polycentricity and monocentricity. In south-east England, for example, at the scale of daily commuting flows of greater than 3500, the region is relatively monocentric (centred on London) while at the scale of daily commuting flows of between 300 and 3500, the region is relatively polycentric (Hall *et al.*, 2006a).

However, no matter what scale is in question, the general approach to using the above techniques will be to identify the group of settlements about which we want to find out and then to apply the techniques to those settlements (worked examples may be found below). In other words, the choice of settlements to be studied must be made in advance and it follows that these techniques are not intended to (and nor can they) define a polycentric urban region, functional urban region, city-region or any other type of region, although they may well assist in that process.

The definitions presented here are also scalable, that is, they can be applied at *any* scale: they could be used to calculate values for functional polycentricity amongst any nodes of roughly the same size, be they village shops, small towns, large businesses or nation-states. The important thing is that the nodes in the network under examination are of approximately the same scale. Thus, while it would therefore be possible, in principle, to compare the polycentricity of a network of regional business links with that of a network of local business links, the disparate nature of two such networks would actually make such a comparison all but meaningless. By contrast, a comparison of low-level commuting networks amongst small settlements in two different regions would yield comparable results.

Interpreting Indices of Functional Polycentricity

It is also worth making the point that while the theoretical limits set by the definitions of Regional Functional Polycentricity, General Functional Polycentricity and Special Functional Polycentricity are 0 and 1, the real-world range of values can be expected to be smaller. The application of the complementarity modifier in General Functional Polycentricity will tend to reduce the maximum value of P_{GF} still further, reflecting the fact that the functional networks in any given collection are unlikely to complement one another perfectly. Values for commuting in eight regions across Europe, for example, calculated using the earlier, unweighted equation for P_{GF} (equation (9)) were in the range of 0.020 to 0.250 (Hall and Pain, 2006a, p. 52) and, for this reason, values for polycentricity are best calculated and given to three or even four decimal places.

A value of one would only be achieved if all possible linkages between nodes are maximal. To put this in context, we can use the example of a commuting network having a value of 1 for Special Functional Polycentricity for in-commuting. The outcome would be that no one would live and work in the same place. Environmental considerations aside, this would be a planner's nightmare, but this *reductio ad absurdum* does make the point that maximum polycentricity in terms of these definitions may not necessarily be desirable.

Getting the Data

The datasets required to put the definitions of functional polycentricity into practice take the form of matrices and the gathering of these data is not always easy. There is the need to choose a dataset and this choice will be influenced by what is considered important, by what data are available (and how readily) and by how good the data are. In some instances, gathering the appropriate data will be a simple task, leaving little room for misinterpretation or ambiguity. No dataset is perfect and compromises must be made. Commuting data, for example, are readily acquired in the UK from sources such as the Census Interaction Data Service, which enables the user to download origin–destination matrices for a set of nodes and at a scale chosen by that user (Census Interaction Data Service, 2005). Here, the source is known and established, and transparency and internal consistency are easily achieved relative to certain other datasets, discussed below.

Other physical trips, such as shopping trips or journeys for leisure purposes, may be relatively easy to measure through the use of large surveys. Friendship networks are simple to analyse at local scales, but for the (relatively) large areas for which we would be analysing polycentricity, vast quantities of data would be needed, requiring large and expensive surveys. Business networks have been explored with some success by Peter Taylor and colleagues, who use companies' websites to explore the network structure of firms and to situate that structure spatially (Taylor, 2004), and the techniques developed by Taylor were used in the POLYNET project to explore polycentricity in each of the eight mega-city regions under examination (Taylor *et al.*, 2006). However, these techniques are very different from those set out in the current article.

Other data to do with connections between businesses are much harder to find and much less clear-cut. Even leaving aside issues of privacy and willingness to hand over datasets that may contain privileged information, a problem found by Hall and colleagues (Hall *et al.*, 2006b), there is still the problem of how to measure the flows themselves. The classic example is e-mail and the dilemma can be summed up in two simple questions.

—If I send one e-mail to 15 people arranging a meeting, is that the same as 15 telephone calls?

—If I arrange a meeting with one person in the course of a single telephone conversation, is that the equivalent of an exchange of several e-mails with that person to achieve exactly the same purpose?

Bringing in even more modern developments, such as text messaging using mobile telephones, perhaps to confirm either of the meetings arranged above, simply complicates matters further. The plain if inconvenient fact of the matter is that there are no easy answers: the wise researcher will test their method using a pilot study.

If we continue to consider e-mail by way of example, the upper limit of how many e-mails someone can send in a day (assuming they are not bulk e-mails) is bounded by the length and size of those e-mails and the speed of the computer and Internet connection. Such physical limits, of course, do not help to address the dilemma raised in the two questions above, although they may help. The key point, as noted earlier, is to recognise that in certain instances there may not be a single 'right answer'. This means that the basis for the final figure (or matrix), however, derived, must be set out with absolute clarity and that in comparisons of the same function in different regions, the same basis is used throughout for each region. In this way, the basis for the final figure can be argued from a position of transparency, thus making it possible for critiques to be well-considered and like to be compared with like. The method can thus be seen to be internally consistent, transparent, replicable and thus usable as the basis for further development and improvement by other researchers.

In the next section, we shall see how these ideas and techniques may be applied in practice.

Example One: Tuonela

In this section, we shall use the equations set out earlier to go through a simple worked example using four hypothetical settlements (the place names all have musical origins, for readers who wonder). Suppose that we wish to analyse polycentricity within a spatial area called Tuonela, which has four small settlements. Note, incidentally, that the calculations that follow make no explicit reference to the spatial size of Tuonela: Tuonela could be like a county in England, or it could be a large, sparsely populated nation with small, widely separated settlements. We shall analyse Tuonela in terms of just two functions: commuting, which is relatively simple to analyse, and business communication via e-mail, which proves somewhat problematic. We assume that quantities (number of commuters; number of e-mails sent and received) are for a single day.

For the purposes of this exercise, which is to illustrate how the definitions of functional polycentricity can be put into practice, we are assuming that the data are readily available and accurate although, as we saw above, such an assumption is likely to be an optimistic one. Our first task is to set out some basic (invented) statistics about the settlements (Tables 1–3). We can use these figures to calculate values for Special and General Functional Polycentricity for each of the functions.

Commuting In Tuonela

For commuting, we want to analyse only movements between settlements. Therefore we omit the values for those who live and work in the same place (these lie on the leading diagonal) and generate a matrix with

Table 1. Tuonela: basic statistics

Settlement	Population	Working population	Area (ha)
Rockville	100	50	7
Hatful Hollow	75	40	4
Fotheringay	40	20	2
Norwegian Wood	80	40	5

Table 2. Commuting

Residence	Workplace				
	Rockville	Hatful Hollow	Fotheringay	Norwegian Wood	OutDegree
Rockville	35	5	2	8	50
Hatful Hollow	15	20	3	2	40
Fotheringay	3	1	15	1	20
Norwegian Wood	8	1	1	30	40
Indegree	61	27	21	41	

Note: this table includes people who work in their town of residence.

Table 3. Business e-mail

Origin	Destination				
	Rockville	Hatful Hollow	Fotheringay	Norwegian Wood	OutDegree
Rockville	150	30	40	15	235
Hatful Hollow	80	40	2	1	123
Fotheringay	15	12	50	3	80
Norwegian Wood	35	15	32	60	142
Indegree	280	97	124	79	

Note: exchanged per day between different businesses.

a null diagonal. Thus, let

Commuting Matrix M_C

$$= \begin{pmatrix} 0 & 5 & 2 & 8 \\ 15 & 0 & 3 & 2 \\ 3 & 1 & 0 & 1 \\ 8 & 1 & 1 & 0 \end{pmatrix}$$

Adding the figures in each column, we get a total value for in-commuting (indegree) $L_{CI} = 50$. Since the example we are looking at is a completely closed system, this equals the total value for out-commuting (outdegree) $L_{CO} = 50$. Thus

$$L = L_{CI} = L_{CO} = 50$$

The density of the commuting networks for both in-commuting and out-commuting can be calculated using equation (6), which is used for directed, valued relations such as flows. Thus using equation (6), $L = 50$ and $L_{\max} = 130$; therefore

$$\Delta_C = 50/130 = 0.38$$

where, Δ_C is the density of the commuting networks; L is the total value of all the edges in the graph; and $L_{\max}$ is the maximum possible value of all the edges in the graph (calculated for all possible other members of the working population commuting to the settlement with the smallest working population, in this case Fotheringay).

Next, we calculate the standard deviation for in-commuting. This gives a value of $\sigma_{CI} = 8.02$, where σ_{CI} is the standard deviation for in-commuting. We calculate $\sigma_{F\max}$ in accordance with equation (2). So for in-commuting, the maximum indegree for any one node is 26 (Rockville) and thus $\sigma_{CI\max} = 13$. We can therefore say that

$$P_{SF}(in - commuting) = \left(\frac{1 - 8.02}{13}\right) \cdot 0.38$$
$$= 0.15$$

Thus, P_{SF} *(in - commuting)* $= 0.15$.

Tuonela therefore appears not to be very polycentric in terms of in-commuting and if

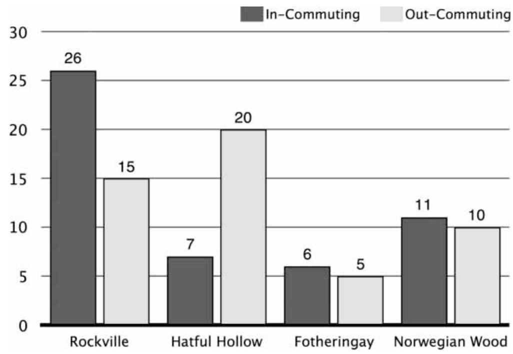

Figure 2. Commuting in Tuonela.

a graph of total in-commuting is generated, we can quickly see that Rockville outstrips the other three settlements in this respect. This can usefully be illustrated with the help of a simple bar chart (Figure 2).

We now turn our attention to out-commuting, and first we calculate the standard deviation σ_{CO} for out-commuting $= 5.59$. We calculate $\sigma_{F\max}$ in accordance with equation (2). For out-commuting, the maximum out-degree for any one node is 20 (Hatful Hollow) and so $\sigma_{CI\max} = 10$. Thus we can say that

$$P_{SF}(out - commuting)$$

$$= (1 - 5.59/10) \cdot 0.38 = 0.17$$

We can see that Tuonela appears to be slightly more polycentric in terms of out-commuting than it is in terms of in-commuting. If a bar chart of total out-commuting is generated, we can quickly see that Hatful Hollow and Rockville outstrip the other two settlements in this respect (Figure 2).

We can also calculate the General Functional Polycentricity for commuting, in accordance with equation (11). First, we calculate the value for the complementarity modifier Φ as defined in equation (9). Thus

$$\Phi = 1 - \sigma\left(\frac{5.59}{10}, \frac{8.02}{13}\right) = 0.96$$

Thus

$$P_{GF}(commuting) = \frac{0.96 \cdot (0.15 + 0.17)}{2}$$

$$= 0.154$$

$$P_{GF}(commuting) = 0.154$$

We can thus see that, in terms of commuting, Tuonela is not very polycentric, dominated as it is by Rockville. We now go on to analyse business e-mail transactions.

Business E-mail in Tuonela

Business communication via e-mail is problematic compared with commuting, for the reasons previously discussed. We shall use exactly the same techniques that we used for the commuting values. First then, since we wish to analyse only the exchanges between settlements, we must generate a matrix with a null diagonal. Thus let

$$\text{E-mail matrix } M_E = \begin{pmatrix} 0 & 30 & 40 & 15 \\ 80 & 0 & 2 & 1 \\ 15 & 12 & 0 & 3 \\ 35 & 15 & 32 & 0 \end{pmatrix}$$

The total value for e-mail indegree $L_{EI} = 280$ and since we are assuming that only single e-mails (i.e. bulk e-mails are not counted) are sent within Tuonela, we can say that the total value for outdegree $L_{EO} = 280$. Thus

$$L = L_{EI} = L_{EO} = 280$$

Calculating the density is more difficult for e-mail exchanges than for commuting. It must be calculated by reference to an empirically derived value, since there is in principle no theoretical maximum for L. In Tuonela, those people who work send out a total of 280 e-mails per day to the other settlements, an average of just under 2 business e-mails per day per working person. Here then, we shall assume that a plausible maximum number of e-mails per day per person may be twice that number; that is, 4 e-mails per day sent by one person, or a total of 600 e-mails per day for all of Tuonela.

The density of the commuting networks for both e-mail indegree and e-mail outdegree can be calculated using equation (6). Thus $L = 280$ and $L_{max} = 600$ and therefore

$$\Delta_E = 280/600 = 0.47$$

where, Δ_E is the density of the business e-mail exchange networks; L is the total value of all the edges in the graph; and L_{max} is an empirically derived maximum possible value of all the edges in the graph.

Next, we calculate the standard deviation for e-mail indegree. Thus $\sigma_{EI} = 39.96$ where σ_{EI} is the standard deviation for e-mail indegree. We calculate $\sigma_{F\max}$ in accordance with equation (2) and so, for e-mail indegree, the maximum indegree for any one node is 130 (Rockville) and therefore $\sigma_{EI\max} = 65$. Thus, we can say that

$$P_{SF}(e\!-\!mails\ received) = (1 - 39.96/65) \cdot 0.47 = 0.18$$

Thus Tuonela appears not to be very polycentric in terms of receipt of business e-mails (e-mail indegree) and we can see from the bar chart of e-mail indegree and e-mail outdegree that once again, Rockville is the dominant settlement (Figure 3).

We now turn our attention to business e-mails sent (e-mail outdegree) and first we calculate the standard deviation for e-mail outdegree. Thus $\sigma_{EO} = 23.12$ where σ_{EO} is the standard deviation for e-mail outdegree.

We calculate $\sigma_{F\max}$ in accordance with equation (2) and so, for e-mail outdegree, the maximum outdegree for any one node is 85 (Rockville) and thus $\sigma_{EO\max} = 42.5$. Thus we can say that

$$P_{SF}(e\!-\!mails\ sent) = (1 - 23.12/42.5) \cdot 0.47 = 0.21.$$

Tuonela is slightly more functionally polycentric in terms of e-mail sent than it is in terms of e-mail received, a finding which is confirmed by comparing the graph of indegree with that of outdegree (Figure 3).

We can also calculate the General Functional Polycentricity for e-mail exchange in accordance with equation (11), as with the earlier example of commuting. First of all, we calculate the value for the complementarity modifier Φ as defined in equation (9). Thus

$$\Phi = 1 - \sigma\left(\frac{39.96}{65}, \frac{23.12}{42.5}\right) = 0.95$$

Thus

$$P_{GF}(business\ e\!-\!mail\ exchange) = \frac{0.95 \cdot (0.18 + 0.21)}{2} = 0.189$$

Therefore

$$P_{GF}(e - mail) = 0.189$$

We can thus see that, in terms of e-mail, Tuonela is a moderately polycentric region and we can also see that it is more polycentric in terms of e-mail than in terms of commuting.

Finally, we can generate an overall value of mean General Functional Polycentricity for both commuting and e-mail, using the following equation. From equation (9), let Mean General Functional Polycentricity

$$\overline{P_{GF}} = \frac{\sum_{n=1}^{n}[P_{GF}(N_1, N_2, \ldots N_n)]}{n} \qquad (17)$$

Thus using the figures for General Functional Polycentricity calculated earlier, we can calculate the Mean General Functional

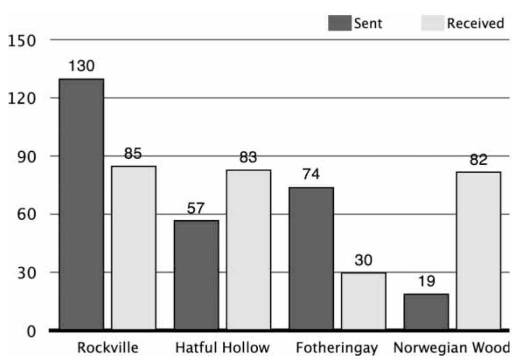

Figure 3. E-mail in Tuonela.

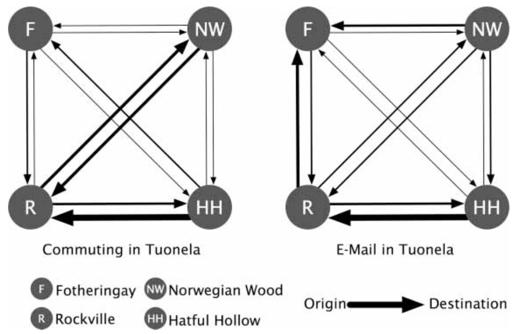

Figure 4. Commuting and e-mail networks in Tuonela.

Polycentricity for Tuonela thus

$$\overline{P_{GF}}(Tuonela) = \frac{(0.189 + 0.154)}{2} = 0.17$$

This figure could be compared with those of other similar regions calculated for the same functions in the way set out above, thus enabling the comparison of functional polycentricity across different regions, for any particular functions or sets of functions, at any scale. The networks can also be visualised as graphs generated from the matrices (Figure 4). These graphs can help us to visualise the overall form of the network, but at the expense of glossing over some of the detail revealed in the calculations.

The question of what to do with this information remains, of course. In Tuonela, it may be that the findings with regard to polycentricity reflect what Tuonela's governing body believe should be happening, or it may be that they do not. If there is a policy that Rockville should be dominant, for example, then we might conclude that the policy is working. If the policy is aiming to promote the two smaller settlements, thus increasing the functional polycentricity of Tuonela as a whole, then time-series data would be needed, to establish whether functional polycentricity with regard to a particular function is increasing or decreasing. As it turns out, this particular issue has a real-life counterpart, the county of Cornwall in the UK, and that is the subject of the second worked example.

Example Two: Cornwall

In this worked example, we shall look at commuting networks in the years 1981, 1991 and 2001 among five districts in Cornwall. We shall also calculate indices of Regional Functional Polycentricity for these five settlements for the three years in question. We shall then look at these findings in the light of a recent spatial policy in Cornwall, Regional Planning Guidance for the South West (DETR, 2002).

Before proceeding, however, an important point needs making. This worked example is purely illustrative, intended solely to show how the techniques described may be put to practical use using real-world data, rather than hypothetical data. It is *not* intended to be a detailed analysis of or commentary on policy in south-west England and nor should it be interpreted as such. Note also that the individual steps of each calculation are not presented; the reader is referred to previous sections for step-by-step examples of how the equations may be applied.

West Cornwall: A Brief Overview of Relevant Policy

The West Cornwall network comprises five districts that may be said to form a 'sub-sub-region' of the Western sub-region set out in Regional Planning Guidance (DETR, 2002, p. 21; see also Figure 5).

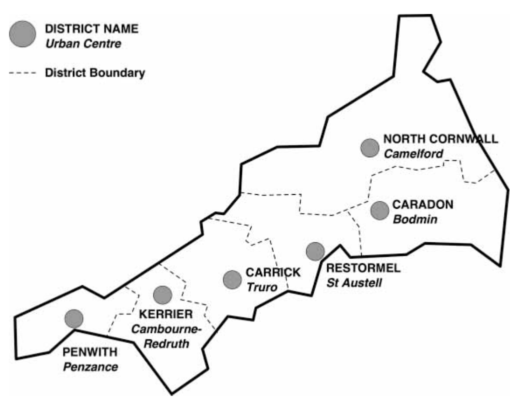

Figure 5. Districts in Cornwall. *Note*: Caradon is not a part of the example network. Scale: approx 1: 200 000 000.

First, we need to explain the choice of network. The network for commuting between districts in England and Wales for values of commuting between 1000 and 10 000 per day includes almost all districts (Census Interaction Data Service, 2005; Green, 2006). However, at the extreme edges of this large network, which covers almost all of England and Wales, there are commuting networks which are not connected to this 'primary' network. The 'West Cornwall network' (WCN) was, in 1981 and 1991, one of these. In 2001, the WCN became an integral part of the 'primary' network, when in terms of commuting flows between 1000 and 10,000, the district of North Cornwall in the WCN joined to the district of Caradon in the primary network (Census Interaction Data Service, 2005; Green, 2006). However, for the purposes of the current paper, we are treating the WCN as a discrete entity, since we are simply illustrating how the formal definitions set out earlier may be put to practical use.

The spatial strategy for the South West region is based around sub-regions which are themselves centred on Principal Urban Areas (PUAs) (DETR, 2002, p.19). This general strategy is supported by a policy to encourage the formation of complementary clusters of smaller centres (DETR, 2002, p. 35). Cornwall does not have any PUAs, but Cambourne and Redruth are recognised as significant small centres, while Truro is the sub-regional shopping and administrative centre (DETR, 2002, p. 36).

Table 4. West Cornwall network, 1981

	Carrick	Kerrier	North Cornwall	Restormel
Carrick	0	183	0	128
Kerrier	364	0	0	0
North Cornwall	0	0	0	143
Restormel	0	0	0	0

Functional Polycentricity in Terms of Commuting

We can of course look at the original origin–destination matrices (Tables 4–6), but bar charts can give us a good overview: we can see for all five of the districts in question that the total volume of in-commuting and out-commuting has increased (Figures 6 and 7). In terms of commuting, then, the system of districts appears to be growing more polycentric. In fact it is not that simple. If we look at values for Special Functional Polycentricity for each of the three years in terms of in-commuting and out-commuting, alongside General

Table 5. West Cornwall network, 1991

	Carrick	Kerrier	North Cornwall	Penwith	Restormel
Carrick	0	229	0	0	150
Kerrier	563	0	0	123	0
Penwith	0	0	0	0	106
North Cornwall	0	175	0	0	0
Restormel	168	0	176	0	0

Table 6. West Cornwall network, 2001

	Carrick	Kerrier	North Cornwall	Penwith	Restormel
Carrick	0	3622	0	0	2106
Kerrier	7543	0	0	1804	0
Penwith	0	0	0	0	1819
North Cornwall	1289	2439	0	0	0
Restormel	3125	0	2952	0	0

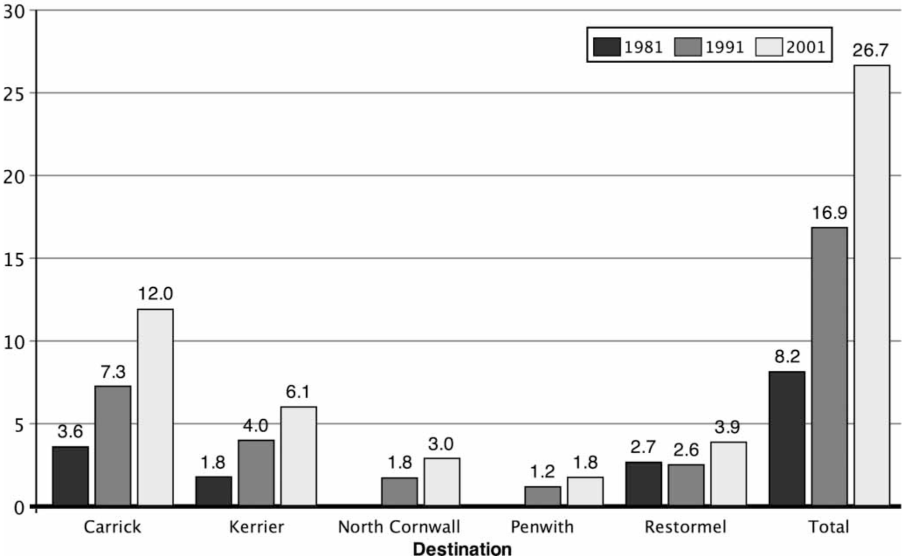

Figure 6. In-commuting in West Cornwall (1000s).

Functional Polycentricity for commuting (Figure 8), we can see that, while all three indices rose from 1981 to 1991, they fell again from 1991 to 2001. This may seem odd, especially in view of the fact that the commuting networks for 1981, 1991 and 2001 also suggest increasing connectivity (Figures 9–11).

If we look at the density of the networks (Figure 12), we find an answer. Although through the incorporation of Penwith the network becomes more evenly balanced over time, in doing so it also becomes less dense and thus the values for functional polycentricity decline very slightly.

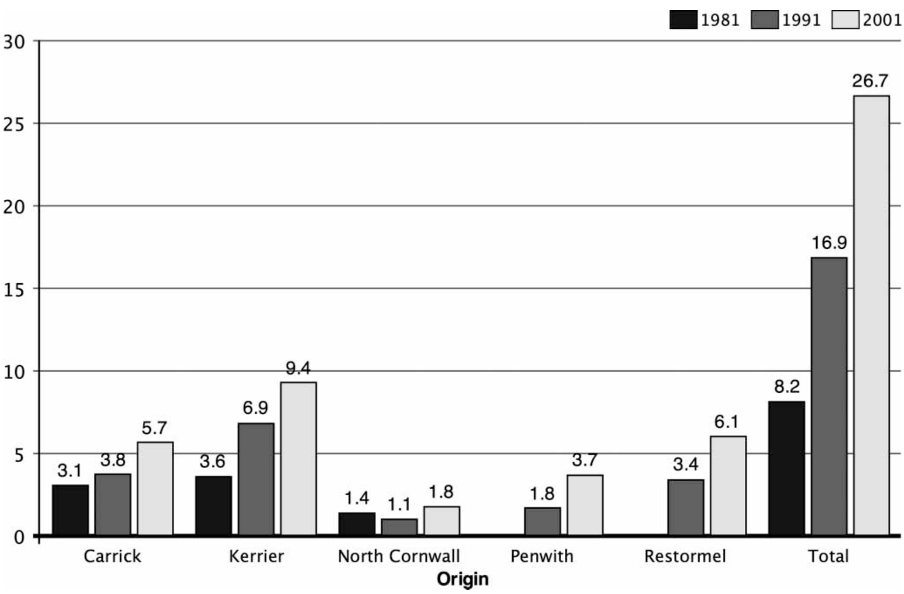

Figure 7. Out-commuting in West Cornwall (1000s).

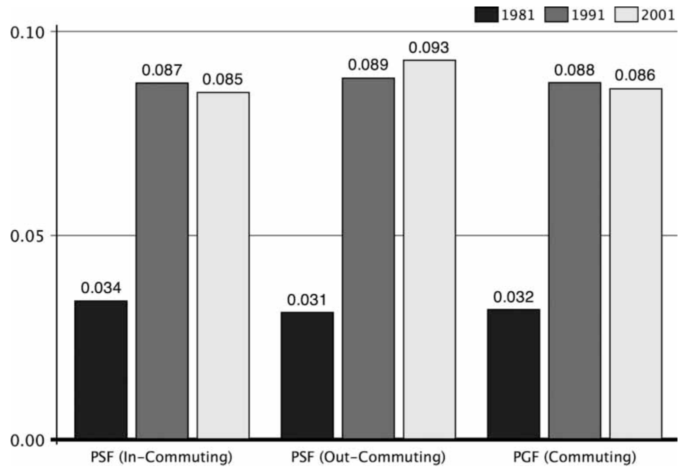

Figure 8. Functional polycentricity in West Cornwall.

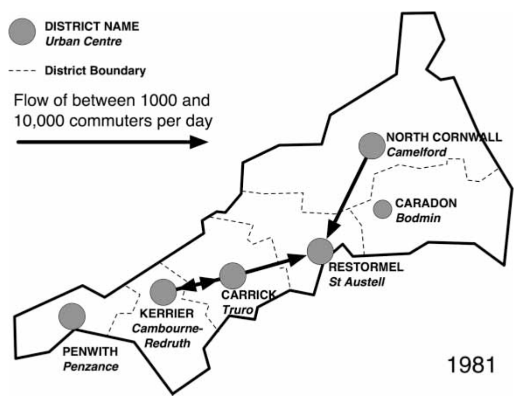

Figure 9. Commuting flows, 1981. Scale: approx 1: 200 000 000.

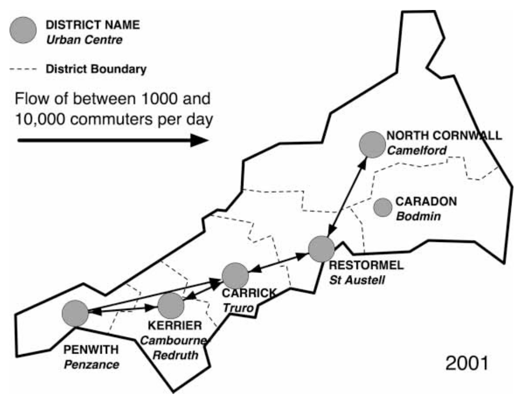

Figure 11. Commuting flows, 2001. Scale: approx 1: 200 000 000.

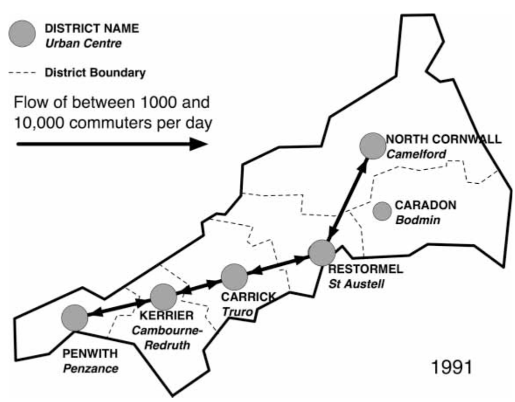

Figure 10. Commuting flows, 1991. Scale: approx 1: 200 000 000.

Regional Functional Polycentricity in West Cornwall

Finally, we come to the question of how R_{PF}, the index of Regional Functional Polycentricity, has changed over time. A quick look at the geography of West Cornwall tells us that, in terms of spatial topography, a reasonable assumption would be that the linear nature of the region would tend to reduce the value of this index, since the standard deviation of distance between settlements is relatively large compared with, say, a five-node network where the nodes are distributed equally around a circle.

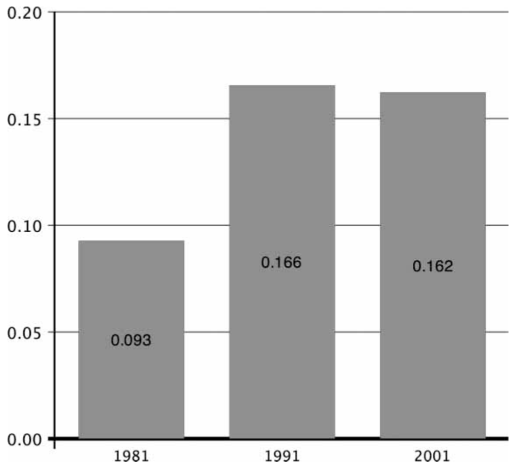

Figure 12. Network density in West Cornwall.

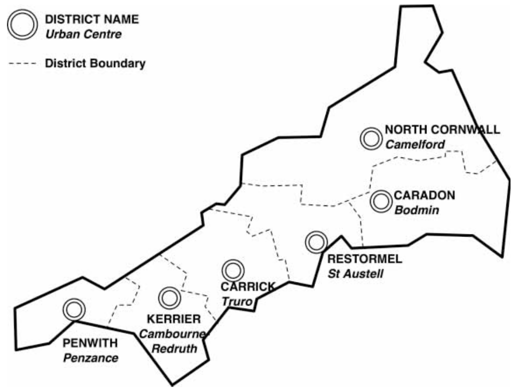

Figure 13. Urban centres in Cornwall. Scale: approx 1: 200 000 000.

For the purposes of this illustrative example, we shall use the values of General Functional Polycentricity for commuting, calculated earlier for 1981, 1991 and 2001, and we shall thus see how the index of Regional Functional Polycentricity has changed over time (since it is a simple multiplier, it will in this instance simply mirror the trajectory of General Functional Polycentricity). We shall calculate the distances between the primary settlements in each district as defined by Regional Planning Guidance (DETR, 2002). In effect, we are treating each district as a proxy for its primary settlement (Figure 13).

First, we calculate topographical polycentricity, P_T. The value of $d_{ij\max}$ is predetermined since we have a defined list of settlements to analyse. We can see the distances, which are very approximate 'as the crow flies' distances between these settlements, in the matrix (Table 7).

From the matrix (Table 7), we can see that the value of $d_{ij\max}$ is 90 km between Penzance (Penwith district) and Camelford (North Cornwall district). Thus

$$\sigma_T = 23.13 \text{ and } \sigma_{T\,\max} = 63.64$$

Applying equation (14), we have

$$P_T = \left(\frac{1 - 23.13}{63.64}\right) = 0.64$$

Following equation (16) we multiply values for P_{GF} for 1981, 1991 and 2001 by P_T to generate values for R_{GF} (commuting 1981, 1991, 2001). These values are shown in the histogram (Figure 14).

But if Penzance is 90 km from Camelford, should we be thinking of all of these settlements as comprising a polycentric urban region? We can make use of equation (13) and attempt to set limits to d_{ij}. The mean distance $\overline{D}$ between all five settlements in the WCN is 46 km, and the standard deviation $\sigma_T = 23$ km. Following equation (13), this gives us a values for $d_{ij(\max)}$

$$d_{ij(\max)} = 46 + 23 = 69 \text{ km}$$

Table 7. West Cornwall network: approximate distances 'as the crow flies' (km)

	Carrick	Kerrier	North Cornwall	Penwith	Restormel
Carrick	—	18	52	42	23
Kerrier	—	—	68	24	42
North Cornwall	—	—	—	—	65
Penwith	—	—	—	90	33
Restormel	—	—	—	—	—

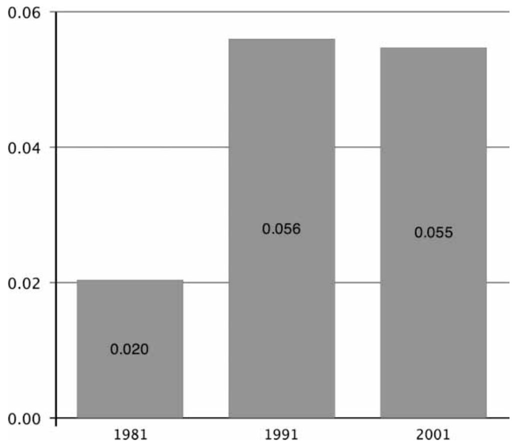

Figure 14. Indices of Regional Functional Polycentricity in West Cornwall.

This value means that either Penzance (Penwith district) or Camelford (North Cornwall district) cannot be included within a single West Cornwall PUR, although we could, in principle, argue that there are in effect two overlapping PURs in West Cornwall, both of which are centred on Truro (Carrick district) and Cambourne/Redruth (Kerrier district). However, given that Camelford is 68 km from Cambourne/Redruth and thus barely within the threshold, we could argue that, in terms of the definitions and analysis set out here, a PUR comprising the four western-most districts of Cornwall is actually the more realistic conceptualisation of the situation 'on the ground'. The corollary is the assertion that a PUR excluding Penzance (Penwith district) and including Camelford (North Cornwall district), while it exists in theory, is a less plausible interpretation of reality.

We can test these assertions using the techniques set out earlier. Thus for the Penwith/Kerrier/Carrick/Restormel PUR (PUR-West), we have

$$d_{ij(\max)}(\text{PUR} - \text{West}) = 65\,km$$
$$\text{(Penwith/Restormel)}$$
$$\sigma_T(\text{PUR} - \text{West}) = 17.6$$
$$\sigma_{T\max}(\text{PUR} - \text{West}) = 45.96$$

and thus

$$P_T(\text{PUR} - \text{West}) = (1 - 17.6/45.96) = 0.62.$$

For the Kerrier/Carrick/Restormel/North Cornwall PUR (PUR-East), we have

$$d_{ij(\max)}(\text{PUR} - \text{East}) = 68\,km$$
$$\text{(Kerrier/North Cornwall)}$$
$$\sigma_T(\text{PUR} - \text{East}) = 18.7$$
$$\sigma_{T\max}(\text{PUR} - \text{East}) = 48.1$$

and thus

$$P_T(\text{PUR} - \text{East}) = (1 - 18.7/48.1) = 0.61$$

Thus the Penwith/Kerrier/Carrick/Restormel PUR is slightly more topographically polycentric than the Kerrier/Carrick/Restormel/North Cornwall PUR. However, if we calculate values for the indices of Regional Functional Polycentricity for each of the four-district PURs, we can see that in fact the PUR-East is consistently more functionally polycentric than the PUR-West (Figure 15).

Findings Viewed in the Light of Policy

Intriguing though all this may be, the question remains of how such insights can help policy-makers. In the case of the WCN, we can turn to the Regional Planning Guidance for the South West of England (DETR, 2002) and see how the aims of the policy compare with the dynamics that we have analysed above, but first, we briefly summarise the findings of this illustrative analysis.

—General Functional Polycentricity measured in terms of commuting among the five districts shows an increase in P_{GF} from 1981 to 1991, and then a slight decline from 1991 to 2001.

—Network density shows the same pattern and we can attribute this to the fact that the district of Penwith joined the network

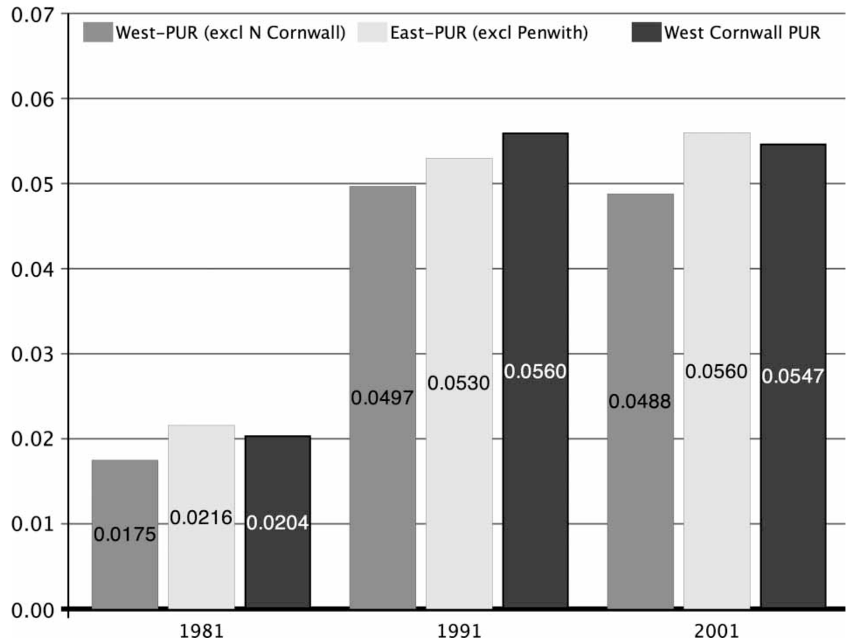

Figure 15. Indices of Regional Functional Polycentricity in two West Cornwall PURs.

in 1991, making it more topographically polycentric, but reducing the density of the network and thus making the network as a whole slightly less functionally polycentric.

—Calculations of Regional Functional Polycentricity reveal that a four-node PUR comprising North Cornwall, Restormel, Carrick and Kerrier has consistently been more functionally polycentric over time than a four node PUR comprising Restormel, Carrick, Kerrier and Penwith.

One of the key policy mechanisms of the South West Regional Planning Guidance is the notion of Principal Urban Areas (PUAs) and sub-regions (DETR, 2002). Cornwall as whole comprises a sub-region, but has no designated PUA. However, Truro and Cambourne–Redruth are pinpointed as foci for economic growth (DETR, 2002, p. 37). The notion of polycentricity is referred to just twice (pp. 10 and 21), but the notion that increased connectivity can contribute to economic growth is acknowledged, as is the role that the sub-regional centres can play in encouraging wider economic growth in a region that is notable for its peninsularity.

What we find from the calculations is that, in terms of commuting, the extreme west of Cornwall is becoming more connected and, importantly, more balanced too. In other words, if the goal is to make western Cornwall more polycentric, then, in terms of commuting, we can reasonably say that that goal is being achieved.

Discussion

Polycentricity as a general concept is being used to inform spatial policy across Europe and yet, nearly a decade on from its inclusion in the ESDP, it has still not been formally defined in terms of one of the most important issues it is meant to address: the ways in which people and settlements interact with one another. This paper has attempted to throw some light on some of these issues by using techniques originating in formal social network analysis, to derive formal definitions of functional polycentricity that can be used to analyse real-world situations. A brief overview of the limited but growing literature on polycentricity found that, although the concept has been used in both an analytical context and as a normative concept, there have been very few attempts to define polycentricity using formal, in other words mathematical, techniques. Those that do exist can be made to break down in certain situations.

This paper has attempted to fill that gap through the use of long-established techniques from social network analysis to develop a formal definition of polycentricity. This definition adopts the position that a collection of nodes, be they cities, small businesses or people, must be functionally connected and balanced if they are to be considered a system. The definition thus considers both the density and balance of the network. The definition has two facets. One definition is called Special Functional Polycentricity and produces a value of between one and zero for a network of settlements in terms of a single specified function. Special Functional Polycentricity was then used as the basis for a second definition, General Functional Polycentricity, which produces a value of between one and zero for a network of settlements in terms of several specified functions. General Functional Polycentricity also works for systems of nodes that may be 'multiple monocentric systems', where each node specialises in a particular function, but the nodes complement one another so that taken as whole, they form a coherent system. The definition was then extended to include the notion of spatial topography and an index of Regional Functional Polycentricity was derived.

Values for functional polycentricity can be derived for any type of functional linkage between nodes: thus values could be derived for, say, business connections, commuting, leisure travel and e-mail traffic between businesses. These values could then be compared. Finally, it is important that the scale of the networks being compared is similar. These ideas were then illustrated using worked examples.

However, this network-analysis-based method of establishing functional polycentricity is not without its weaknesses. In particular, measuring networks of e-mail (or similar) connections, highlights the fact that in certain situations some values within the calculations must be empirically derived from a range of possible values; the derivation itself, however, is a matter of judgement. For example, is an exchange of e-mails required to arrange a single meeting equivalent to the single telephone call that does the same thing? What is important here is that the way in which the choice of value is reached is set out clearly and that, once a method of deriving that value has been arrived at, it is used consistently in comparisons. In terms of measuring Regional Functional Polycentricity, the question of how far settlements can be from one another is also a rather open one.

It should also be pointed out that the theoretical limits of functional polycentricity (one and zero) are just that; theoretical limits. Thus for a two-town network, a value of one would mean that everyone in town A commutes to town B, and everyone in town B commutes to town A: no one would live and work in their home town. Values calculated for eight regions in Europe as part of the POLYNET project, suggest that real-world values are likely to lie between 0.02 and 0.25, and values of functional polycentricity should be interpreted with this in mind. It should also be borne in mind that since network density is a factor, two similar values of functional polycentricity may hide important differences in network morphology.

The methods set out in this paper can therefore only give us an overview of the functional morphology of a region (in its broadest sense) and would have to be augmented with qualitative research that explores the choices that result in a particular functional morphology (of which functional polycentricity is but one).

Nonetheless, the techniques set out herein do have distinct advantages. The method is scalable, from the level of the individual person, to the nation-state and beyond. Using these techniques, functional polycentricity can be compared across different scales and for different functions or sets of functions, both within and across those scales. These techniques are genuinely versatile.

The key point in all these issues, where the choice of data is made by the researcher, is that judgement must be exercised, with all that that implies. The advantage of using mathematical techniques such as those set out above to explore fuzzy notions such as polycentricity is only partly to do with the fact that precise answers can be derived. Just as important is that the means by which one arrives at those answers, even if flawed, are set out clearly and transparently, and so offer common ground for continued, and hopefully constructive, debate.

However, as Hall and Pain (2006b) have pointed out, much remains to be done: spatial and functional complemetarities, the role of government, a lack of in-depth analysis, implications for policy and sustainability form part of the continuing research agenda.

The definitions set out in the present article are just a beginning. Network analysis, which is based on graph theory and matrices, is an enormously versatile way of understanding social systems underpinned by a rich variety of techniques, only some of which have been used here. The ways in which information is transferred within and between networks of cities (for a discussion of this issue with regard to people, see for example Burt, 2004) is one avenue that is surely worth exploring, given that polycentricity's normative role appears to be the fostering of co-operation, and in light of the fact that information transfer so often proves crucial to a region's economic success (Saxenian, 1994; Hall, 1998; Landry, 2000). The techniques set out here, it is hoped, will assist in that process.

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